LESSON 3.1c

Solving Quadratics by Graphing and by Square Roots

Today you will:

- Solve quadratic functions by graphing
- Solve quadratic functions by finding square roots
- Practice using English to describe math processes and equations

Core Vocabulary:

- quadratic equation in one variable, p. 94
- root of an equation, p. 94
- Previous
    - properties of square roots
    - rationalizing the denominator

What does it mean to solve $x + 2 = 3$?

- find the single value of x that makes the equation "work" or be true…there is only 1 variable

What does it mean to solve $y = x + 2$?

- find ***all*** the values of x that makes the equation "work" or be true ***AND*** the corresponding $y's$
- that is ****A LOT**** of values! The domain and range are both all real numbers…there are 2 variables

What does it mean to solve $y = x^2 + x - 2$?

- find all the values of x that makes the equation "work" or be true ***AND*** the corresponding $y's$
- again, a lot of values …
- typically when we think about "solving" a quadratic we are interested in the ***defining*** values for the equation
- What do you think are ***defining*** values for a quadratic?
  - vertex
  - x-intercepts
  - y-intercepts

So how do we define "solving a quadratic?"

- By finding it's *x*-intercepts.

- Definition:
  - ***<u>Roots of an equation</u>***: the *x*-intercepts
  - To find the roots of a quadratic means find it's *x*-intercepts

- How can we find the **roots of an equation** (*x*-intercepts)?

  - One way is by graphing the equation on your graphing calculator
    - Hit **Y=** and enter the equation
    - Hit **GRAPH** to see the graph
    - Hit **TABLE** (2^{nd} then GRAPH) to see the table of values
    - Use the up and down arrows to move through the table and find the *x*-intercepts (where *y* is zero)
    - On paper double-check your answer (plug the *x* values in and verify you get zero)

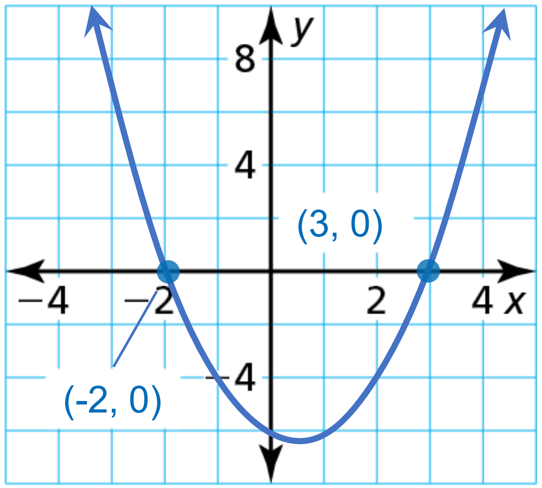
Solve each equation by graphing.

a. $x^2 - x - 6 = 0$

b. $-2x^2 - 2 = 4x$

SOLUTION

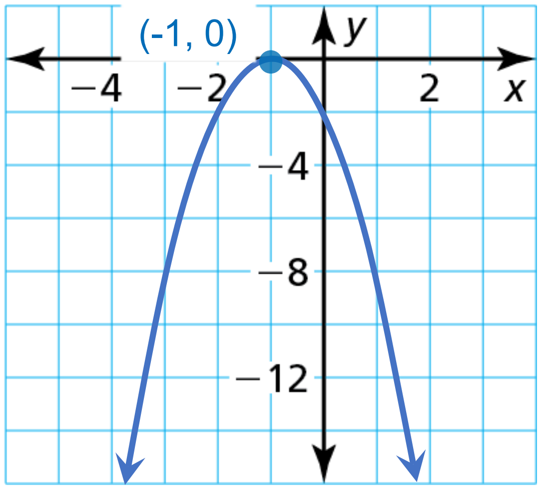
a. The equation is in standard form. Graph the related function $y = x^2 - x - 6$.



The x-intercepts are -2 and 3.

The solutions, or roots, are $x = -2$ and $x = 3$.

b. Add $-4x$ to each side to obtain $-2x^2 - 4x - 2 = 0$. Graph the related function $y = -2x^2 - 4x - 2$.



The x-intercept is -1.

The solution, or root, is $x = -1$.

Check

$$x^2 - x - 6 = 0$$
$$(-2)^2 - (-2) - 6 \stackrel{?}{=} 0$$
$$4 + 2 - 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$$x^2 - x - 6 = 0$$
$$3^2 - 3 - 6 \stackrel{?}{=} 0$$
$$9 - 3 - 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

How can we find the **roots of an equation** (*x*-intercepts)?

1. By graphing the equation on your graphing calculator (we just did this)

2. Algebraically (what we are going to do next)

    - Get the equation into standard form and set equal to zero: $ax^2 + bx + c = 0$
    - …then solve for *x!*

    - What are we doing when we do this?
        - In $ax^2 + bx + c = 0$ what is the value of y?
        - When $y = 0$ we are finding the x-intercepts! We are finding the ***roots of the equation.***

We are going to learn a few different methods of solving a quadratic algebraically:

- using square roots (L3.1 today)
- by factoring / "finding the zeros" of the function (L3.1 tomorrow)
- completing the square (L3.3)
- using the quadratic function (L3.4)

Square roots:

1. A **square root** of a number is a value that, when multiplied by itself, gives the number.
2. Example: $4 \times 4 = 16$, so a **square root** of 16 is 4.

Why does it say “so **A** square root of 16 is 4”???
Right! Because there is another number that works here: -4

NOTE: any positive real number has two **square roots**, one positive and one negative.

Why does it say “any **POSITIVE** real number”???
Because you can’t take the square root of a negative number.

Why?
Because you can’t find a number which multiplied by itself gives a negative number.
A negative times a negative gives a positive.

Solving with square roots:

$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = \sqrt{5 \cdot 5}$

$x = \pm 5$

$\sqrt{aa} = \pm a$

$x^2 = 18$

$\sqrt{x^2} = \sqrt{18}$

$x = \pm\sqrt{18}$ simplify

$x = \pm\sqrt{9 \cdot 2}$

$x = \pm\sqrt{9}\sqrt{2}$

$x = \pm 3\sqrt{2}$

Product Property of Square Roots

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$x^2 = \frac{4}{3}$

$\sqrt{x^2} = \sqrt{\frac{4}{3}}$

$x = \pm\sqrt{\frac{4}{3}}$ simplify

$x = \pm\frac{\sqrt{4}}{\sqrt{3}}$

$x = \pm\frac{2}{\sqrt{3}}$ rationalize the denominator

$x = \pm\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{3}}{3}$

Quotient Property of Square Roots

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

Solve each equation using square roots.

a. $4x^2 - 31 = 49$ **b.** $3x^2 + 9 = 0$ **c.** $\frac{2}{5}(x + 3)^2 = 5$

SOLUTION

a. $4x^2 - 31 = 49$ Write the equation.

$4x^2 = 80$ Add 31 to each side.

$x^2 = 20$ Divide each side by 4.

$x = \pm\sqrt{20}$ Take square root of each side.

$x = \pm\sqrt{4} \cdot \sqrt{5}$ Product Property of Square Roots

$x = \pm 2\sqrt{5}$ Simplify.

The solutions are $x = 2\sqrt{5}$ and $x = -2\sqrt{5}$.

b. $3x^2 + 9 = 0$ Write the equation.

$3x^2 = -9$ Subtract 9 from each side.

$x^2 = -3$ Divide each side by 3.

The square of a real number cannot be negative. So, the equation has no real solution.

LOOKING FOR STRUCTURE

Notice that $(x+3)^2 = \frac{25}{2}$ is of the form $u^2 = d$, where $u = x + 3$.

c. $\frac{2}{5}(x+3)^2 = 5$ Write the equation.

$(x+3)^2 = \frac{25}{2}$ Multiply each side by $\frac{5}{2}$.

$x + 3 = \pm\sqrt{\frac{25}{2}}$ Take square root of each side.

$x = -3 \pm \sqrt{\frac{25}{2}}$ Subtract 3 from each side.

$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}}$ Quotient Property of Square Roots

$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$x = -3 \pm \frac{5\sqrt{2}}{2}$ Simplify.

STUDY TIP

Because $\frac{\sqrt{2}}{\sqrt{2}} = 1$, the value of $\frac{\sqrt{25}}{\sqrt{2}}$ does not change when you multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

The solutions are $x = -3 + \frac{5\sqrt{2}}{2}$ and $x = -3 - \frac{5\sqrt{2}}{2}$.

Homework

Pg 99, #3-24